AMENDMENTS TO THE CLAIMS

This listing of claims will replace all prior versions, and listings, of claims in the application:

Listing of Claims:

1. (Currently amended) A method for using a computer system to solve a system of nonlinear equations specified by a vector function, **f**, wherein $\mathbf{f}(\mathbf{x}) = \mathbf{0}$ represents $f_1(\mathbf{x}) = 0, f_2(\mathbf{x}) = 0, f_3(\mathbf{x}) = 0, \ldots, f_n(\mathbf{x}) = 0$, wherein **x** is a vector $(x_1, x_2, x_3, \ldots x_n)$, the method comprising:

receiving a representation of an interval vector $\mathbf{X} = (X_1, X_2, \ldots, X_n)$, wherein for each dimension, i, the representation of X_i includes a first floating-point number, a_i, representing the left endpoint of X_i, and a second floating-point number, b_i, representing the right endpoint of X_i;

for each nonlinear equation $f_i(\mathbf{x}) = g(x'_j) - h(\mathbf{x}) = 0$ in the system of equations $\mathbf{f}(\mathbf{x}) = \mathbf{0}$, symbolically manipulating $f_i(\mathbf{x})=0$ within the computer system to solve for any invertible term, $g(x'_j)$, thereby producing a modified equation $g(x'_j)= h(\mathbf{x})$, wherein $g(x'_j)$ ~~can be~~ is analytically inverted to produce an inverse function $g^{-1}(\mathbf{y})$;

substituting the interval vector **X** into the modified equation to produce the equation $g(X'_j) = h(\mathbf{X})$;

solving for $X'_j = g^{-1}(h(\mathbf{X}))$; and

intersecting X'_j with the vector element X_j to produce a new interval vector $\mathbf{X}^+$;

wherein the new interval vector $\mathbf{X}^+$ contains all solutions of the system of equations $\mathbf{f}(\mathbf{x}) = \mathbf{0}$ within the interval vector **X**, and wherein the width of the new interval vector $\mathbf{X}^+$ is less than or equal to the width of the interval vector **X**.

2. (Original) The method of claim 1, further comprised of performing an interval Newton step on **X** to produce a resulting interval vector, **Y**, wherein the point of expansion of the interval Newton step is a point, **x**, within the interval vector **X**, and wherein performing the interval Newton step involves evaluating **f**(**x**) using interval arithmetic to produce an interval result $\mathbf{f}^{I}(\mathbf{x})$.

3. (Original) The method of claim 2, further comprising:

evaluating a first termination condition, wherein the first termination condition is TRUE if,

zero is contained within $\mathbf{f}^{I}(\mathbf{x})$,

J(**x**,**X**) is regular, wherein **J**(**x**,**X**) is the Jacobian of the function **f** evaluated as a function of **x** over the interval vector **X**, and

Y contained within **X**; and

if the first termination condition is TRUE, terminating and recording $\mathbf{X} = \mathbf{X} \cap \mathbf{Y}$ as a final bound.

4. (Original) The method of claim 3, further comprising determining if **J**(**x**,**X**) is regular by computing a pre-conditioned Jacobian, **M**(**x**,**X**) = **BJ**(**x**,**X**), wherein **B** is an approximate inverse of the center of **J**(**x**,**X**), and then solving for the interval vector **Y** that contains the value of **y** that satisfies **M**(**x**,**X**)(**y**-**x**) = **r**(**x**), where **r**(**x**) = -**Bf**(**x**).

5. (Original) The method of claim 4, further comprising applying term consistency to **Bf**(**x**) = 0.

6. (Original) The method of claim 1, wherein if no termination condition is satisfied, the method further comprises returning to perform an interval Newton step on the interval vector **Y**.

7. (Original) The method of claim 6, wherein returning to perform the interval Newton step on the interval vector **Y** can involve splitting the interval vector $\mathbf{X}=\mathbf{Y} \cap \mathbf{X}$.

8. (Original) The method of claim 2, further comprising:

evaluating a second termination condition;

wherein the second termination condition is TRUE if a function of the width of the interval vector **X** is less than a pre-specified value, ε_X, and the absolute value of the function, **f**, over the interval vector **X** is less than a pre-specified value, ε_F; and

if the second termination condition is TRUE, terminating and recording **X** as a final bound.

9. (Original) The method of claim 1, wherein for each term, $g(x_j)$, that can be analytically inverted within the equation $f_i(\boldsymbol{x}) = 0$, the method further comprises:

setting $X_j = X_j^+$ in **X**; and

repeating the process of symbolically manipulating, substituting, solving and intersecting to produce the new interval vector X_j^+.

10. (Original) The method of claim 1, wherein symbolically manipulating $f_i(\boldsymbol{x})=0$ involves selecting the invertible term $g(x_j)$ as the dominating term of the function $f_i(\boldsymbol{x}) = 0$ within the interval vector **X**.

11. (Currently amended) A computer-readable storage medium storing instructions that when executed by a computer cause the computer to perform a method for using a computer system to solve a system of nonlinear equations specified by a vector function, **f**, wherein $\mathbf{f}(\mathbf{x}) = \mathbf{0}$ represents $f_1(\mathbf{x}) = 0, f_2(\mathbf{x}) = 0, f_3(\mathbf{x}) = 0, \ldots, f_n(\mathbf{x}) = 0$, wherein **x** is a vector $(x_1, x_2, x_3, \ldots x_n)$, the method comprising:

receiving a representation of an interval vector $\mathbf{X} = (X_1, X_2, \ldots, X_n)$, wherein for each dimension, i, the representation of X_i includes a first floating-point number, a_i, representing the left endpoint of X_i, and a second floating-point number, b_i, representing the right endpoint of X_i;

for each nonlinear equation $f_i(\mathbf{x}) = g(x'_j) - h(\mathbf{x}) = 0$ in the system of equations $\mathbf{f}(\mathbf{x}) = \mathbf{0}$, symbolically manipulating $f_i(\mathbf{x})=0$ within the computer system to solve for any invertible term, $g(x'_j)$, thereby producing a modified equation $g(x'_j)= h(\mathbf{x})$, wherein $g(x'_j)$ ~~can be~~ is analytically inverted to produce an inverse function $g^{-1}(\mathbf{y})$;

substituting the interval vector **X** into the modified equation to produce the equation $g(X'_j) = h(\mathbf{X})$;

solving for $X'_j = g^{-1}(h(\mathbf{X}))$; and

intersecting X'_j with the vector element X_j to produce a new interval vector $\mathbf{X}^+$;

wherein the new interval vector $\mathbf{X}^+$ contains all solutions of the system of equations $\mathbf{f}(\mathbf{x}) = \mathbf{0}$ within the interval vector **X**, and wherein the width of the new interval vector $\mathbf{X}^+$ is less than or equal to the width of the interval vector **X**.

12. (Original) The computer-readable storage medium of claim 11, wherein the method further comprises performing an interval Newton step on **X** to produce a resulting interval vector, **Y**, wherein the point of expansion of the interval Newton step is a point, **x**, within the interval vector **X**, and wherein

performing the interval Newton step involves evaluating $\mathbf{f}(\mathbf{x})$ using interval arithmetic to produce an interval result $\mathbf{f}^I(\mathbf{x})$.

13. (Original) The computer-readable storage medium of claim 12, wherein the method further comprises:

evaluating a first termination condition, wherein the first termination condition is TRUE if,

zero is contained within $\mathbf{f}^I(\mathbf{x})$,

$\mathbf{J}(\mathbf{x},\mathbf{X})$ is regular, wherein $\mathbf{J}(\mathbf{x},\mathbf{X})$ is the Jacobian of the function $\mathbf{f}$ evaluated as a function of $\mathbf{x}$ over the interval vector $\mathbf{X}$, and

$\mathbf{Y}$ is contained within $\mathbf{X}$; and

if the first termination condition is TRUE, terminating and recording $\mathbf{X} = \mathbf{X} \cap \mathbf{Y}$ as a final bound.

14. (Original) The computer-readable storage medium of claim 13, wherein the method further comprises determining if $\mathbf{J}(\mathbf{x},\mathbf{X})$ is regular by computing a pre-conditioned Jacobian, $\mathbf{M}(\mathbf{x},\mathbf{X}) = \mathbf{B}\mathbf{J}(\mathbf{x},\mathbf{X})$, wherein $\mathbf{B}$ is an approximate inverse of the center of $\mathbf{J}(\mathbf{x},\mathbf{X})$, and then solving for the interval vector $\mathbf{Y}$ that contains the value of $\mathbf{y}$ that satisfies $\mathbf{M}(\mathbf{x},\mathbf{X})(\mathbf{y}-\mathbf{x}) = \mathbf{r}(\mathbf{x})$, where $\mathbf{r}(\mathbf{x}) = -\mathbf{B}\mathbf{f}(\mathbf{x})$.

15. (Original) The computer-readable storage medium of claim 14, wherein the method further comprises applying term consistency to $\mathbf{B}\mathbf{f}(\mathbf{x}) = 0$.

16. (Original) The computer-readable storage medium of claim 11, wherein if no termination condition is satisfied, the method further comprises returning to perform an interval Newton step on the interval vector $\mathbf{Y}$.

17. (Original) The computer-readable storage medium of claim 16, wherein returning to perform the interval Newton step on the interval vector **Y** can involve splitting the interval vector $\mathbf{X}=\mathbf{Y} \cap \mathbf{X}$.

18. (Original) The computer-readable storage medium of claim 12, wherein the method further comprises:

evaluating a second termination condition;

wherein the second termination condition is TRUE if a function of the width of the interval vector **X** is less than a pre-specified value, ε_X, and the absolute value of the function, **f**, over the interval vector **X** is less than a pre-specified value, ε_F; and

if the second termination condition is TRUE, terminating and recording **X** as a final bound.

19. (Original) The computer-readable storage medium of claim 11, wherein for each term, $g(x_j)$, that can be analytically inverted within the equation $f_i(\mathbf{x}) = 0$, the method further comprises:

setting $X_j = X_j^{\ +}$ in **X**; and

repeating the process of symbolically manipulating, substituting, solving and intersecting to produce the new interval vector $X_j^{\ +}$.

20. (Original) The computer-readable storage medium of claim 11, wherein symbolically manipulating $f_i(\mathbf{x})=0$ involves selecting the invertible term $g(x_j)$ as the dominating term of the function $f_i(\mathbf{x}) = 0$ within the interval vector **X**.

21. (Currently amended) An apparatus that uses a computer system to solve a system of nonlinear equations specified by a vector function, **f**, wherein

$\mathbf{f}(\mathbf{x}) = \mathbf{0}$ represents $f_1(\mathbf{x}) = 0, f_2(\mathbf{x}) = 0, f_3(\mathbf{x}) = 0, \ldots, f_n(\mathbf{x}) = 0$, wherein $\mathbf{x}$ is a vector $(x_1, x_2, x_3, \ldots x_n)$, the apparatus comprising:

a receiving mechanism that is configured to receive a representation of an interval vector $\mathbf{X} = (X_1, X_2, \ldots, X_n)$, wherein for each dimension, i, the representation of X_i includes a first floating-point number, a_i, representing the left endpoint of X_i, and a second floating-point number, b_i, representing the right endpoint of X_i;

a symbolic manipulation mechanism, wherein for each nonlinear equation $f_i(\mathbf{x}) = g(x'_j) - h(\mathbf{x}) = 0$ in the system of equations $\mathbf{f}(\mathbf{x}) = \mathbf{0}$, the symbolic manipulation mechanism is configured to manipulate $f_i(\mathbf{x})=0$ to solve for any invertible term, $g(x'_j)$, thereby producing a modified equation $g(x'_j)= h(\mathbf{x})$, wherein $g(x'_j)$ ~~can be~~ is analytically inverted to produce an inverse function $g^{-1}(\mathbf{y})$;

a solving mechanism that is configured to,

substitute the interval vector $\mathbf{X}$ into the modified equation to produce the equation $g(X'_j) = h(\mathbf{X})$, and to

solve for $X'_j = g^{-1}(h(\mathbf{X}))$; and

an intersecting mechanism that is configured to intersect X'_j with the vector element X_j to produce a new interval vector $\mathbf{X}^+$, wherein the new interval vector $\mathbf{X}^+$ contains all solutions of the system of equations $\mathbf{f}(\mathbf{x}) = \mathbf{0}$ within the interval vector $\mathbf{X}$, and wherein the width of the new interval vector $\mathbf{X}^+$ is less than or equal to the width of the interval vector $\mathbf{X}$.

22. (Original) The apparatus of claim 21, further comprising an interval Newton mechanism that is configured to perform an interval Newton step on $\mathbf{X}$ to produce a resulting interval vector, $\mathbf{Y}$, wherein the point of expansion of the interval Newton step is a point, $\mathbf{x}$, within the interval vector $\mathbf{X}$, and wherein performing the interval Newton step involves evaluating $\mathbf{f}(\mathbf{x})$ using interval arithmetic to produce an interval result $\mathbf{f}^{1}(\mathbf{x})$.

23. (Original) The apparatus of claim 22, further comprising a termination mechanism that is configured to:

evaluate a first termination condition, wherein the first termination condition is TRUE if,

zero is contained within $\mathbf{f}^{I}(\mathbf{x})$,

$\mathbf{J}(\mathbf{x},\mathbf{X})$ is regular, wherein $\mathbf{J}(\mathbf{x},\mathbf{X})$ is the Jacobian of the function $\mathbf{f}$ evaluated as a function of $\mathbf{x}$ over the interval vector $\mathbf{X}$, and

$\mathbf{Y}$ is contained within $\mathbf{X}$; and to

wherein if the first termination condition is TRUE, the termination mechanism is configured to terminate and recording $\mathbf{X} = \mathbf{X} \cap \mathbf{Y}$ as a final bound.

24. (Original) The apparatus of claim 23, wherein the termination mechanism is configured to determine if $\mathbf{J}(\mathbf{x},\mathbf{X})$ is regular by computing a pre-conditioned Jacobian, $\mathbf{M}(\mathbf{x},\mathbf{X}) = \mathbf{B}\mathbf{J}(\mathbf{x},\mathbf{X})$, wherein $\mathbf{B}$ is an approximate inverse of the center of $\mathbf{J}(\mathbf{x},\mathbf{X})$, and then to solve for the interval vector $\mathbf{Y}$ that contains the value of $\mathbf{y}$ that satisfies $\mathbf{M}(\mathbf{x},\mathbf{X})(\mathbf{y}-\mathbf{x}) = \mathbf{r}(\mathbf{x})$, where $\mathbf{r}(\mathbf{x}) = -\mathbf{B}\mathbf{f}(\mathbf{x})$.

25. (Original) The apparatus of claim 24, wherein the symbolic manipulation mechanism is additionally configured to apply term consistency to $\mathbf{B}\mathbf{f}(\mathbf{x}) = 0$.

26. (Original) The apparatus of claim 21, wherein if no termination condition is satisfied, the apparatus is configured to return to perform an interval Newton step on the interval vector $\mathbf{Y}$.

27. (Original) The apparatus of claim 26, wherein returning to perform the interval Newton step on the interval vector **Y** can involve splitting the interval vector $\mathbf{X}=\mathbf{Y} \cap \mathbf{X}$.

28. (Original) The apparatus of claim 22, wherein the termination mechanism that is configured to:

evaluate a second termination condition;

wherein the second termination condition is TRUE if a function of the width of the interval vector **X** is less than a pre-specified value, ε_X, and the absolute value of the function, **f**, over the interval vector **X** is less than a pre-specified value, ε_F; and

wherein if the second termination condition is TRUE, the termination mechanism is configured to terminate and record **X** as a final bound.

29. (Original) The apparatus of claim 21, wherein for each term, $g(x_j)$, that can be analytically inverted within the equation $f_i(\boldsymbol{x}) = 0$, the apparatus is configured to:

set $X_j = X_j^+$ in **X**; and to

repeat the process of symbolically manipulating, substituting, solving and intersecting to produce the new interval vector X_j^+.

30. (Original) The apparatus of claim 21, wherein symbolically manipulating $f_i(\boldsymbol{x})=0$ involves selecting the invertible term $g(x_j)$ as the dominating term of the function $f_i(\boldsymbol{x}) = 0$ within the interval vector **X**.